A recently asked question

... BACCALAUREATE 2020 : Reser...

PART B

QUESTION B1 ANALYSIS	Page 1/1

Use your calculator in b).

Consider the family of functions f_n defined by

$$f_n(x) = x^n e^{-x}, \quad \text{where } n \in \{2, 3, 4, ...\}.$$

a) Determine the coordinates of the extrema and the points of inflection of the graph of f_4. — 5 marks

The region M is bounded by the graphs of f_2 and f_4, and by the lines ... and $x = 6$.

... the volume of the solid of revolution obtained by rotating the ... x-axis.

Consider the family of functions f_n defined by

$$f_n(x) = x^n e^{-x}, \quad \text{where } n \in \{2, 3, 4, ...\}.$$

a) Determine the coordinates of the extrema and the points of inflection of the graph of f_4. | 5 marks

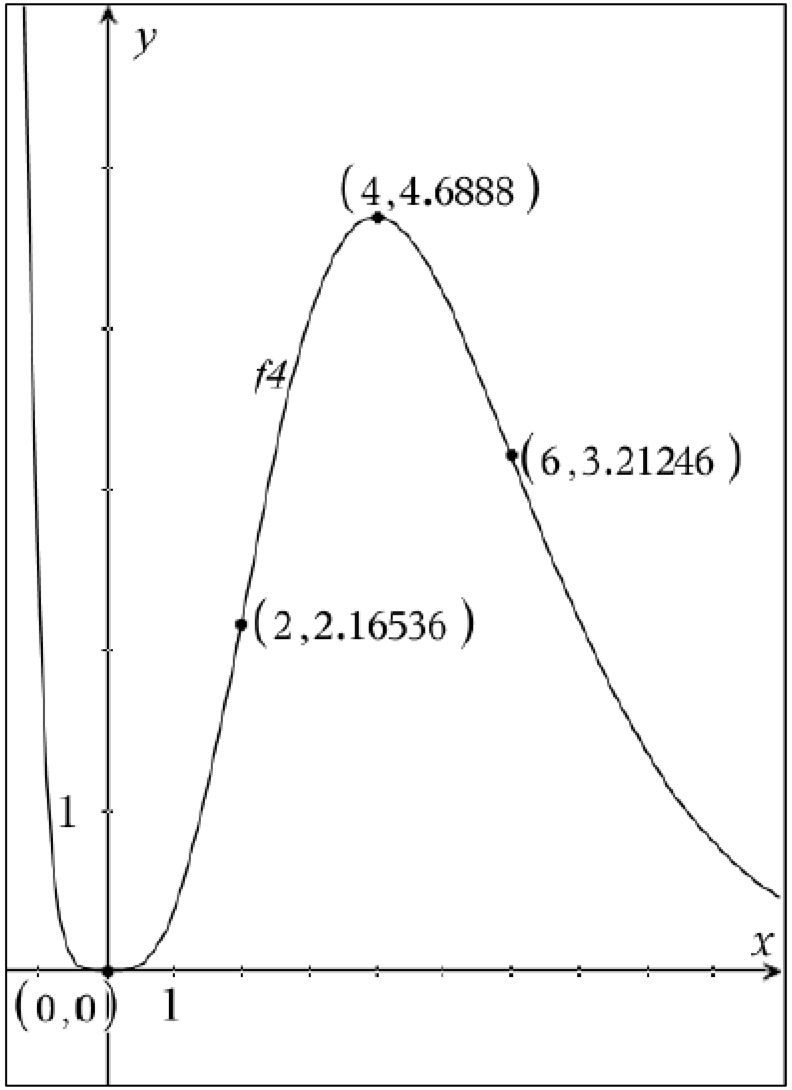
$\text{solve}(f4p(x)=0,x)$ ▸ $x=0$ or $x=4$

$\text{solve}(f4p(x)>0,x)$ ▸ $0<x<4$

$\text{solve}(f4p(x)<0,x)$ ▸ $x<0$ or $x>4$,

$\text{solve}(f4pp(x)=0,x)$ ▸ $x=0$ or $x=2$ or $x=6$ (0,5 P.)

$\text{solve}(f4pp(x)>0,x)$ ▸ $x\neq0$ and $x<2$ or $x>6$ und $\text{solve}(f4pp(x)<0,x)$ ▸ $2<x<6$

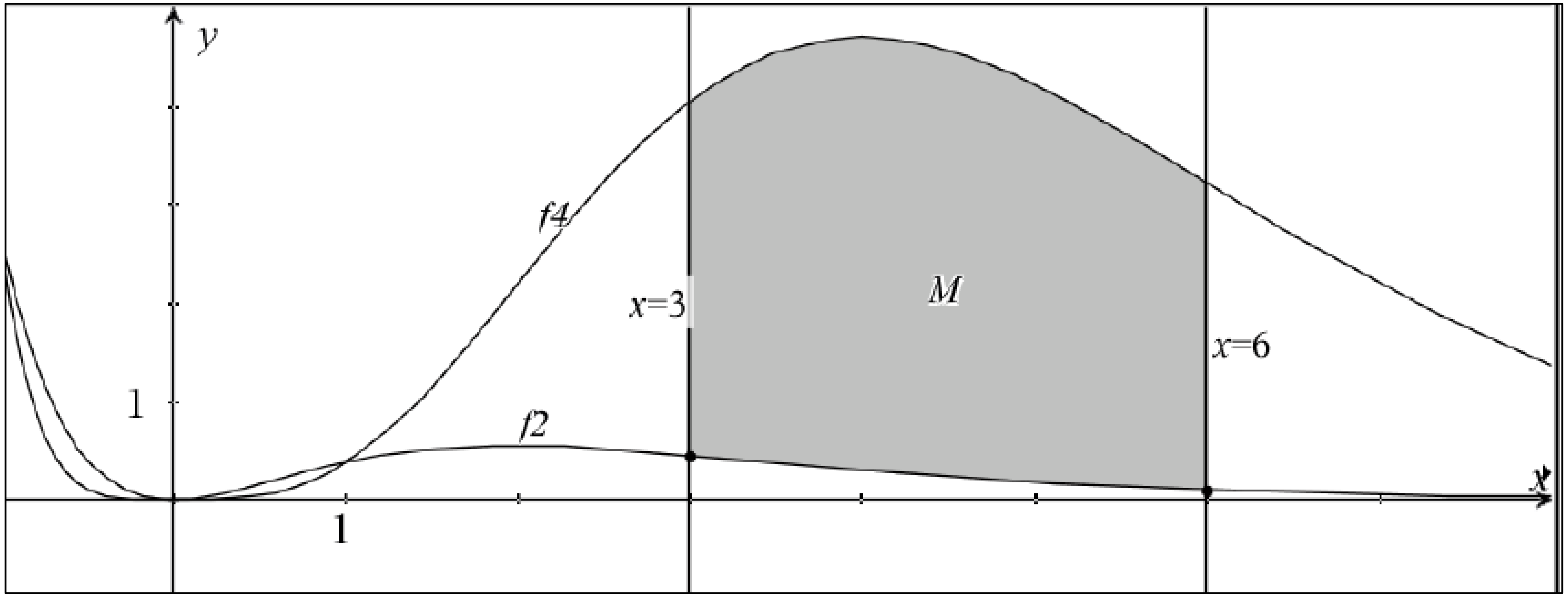


https://www.mentimeter.com/app

b) The region M is bounded by the graphs of f_2 and f_4, and by the lines $x=3$ and $x=6$. Determine the volume of the solid of revolution obtained by rotating the region M about the x-axis.	4 marks	

$$V=\pi\cdot\int_3^6\left|(\mathbf{f4}(x))^2-(\mathrm{f2}(x))^2\right|\,\mathrm{d}x \;\blacktriangleright\; \frac{3\cdot\left(17887\cdot\mathbf{e}^{6}-1324036\right)\cdot\mathbf{e}^{-12}\cdot\pi}{2}$$

$\approx$**170,599.**



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Even more practice

c)	Show that all the curves with equation $y = f_n(x)$ have two points in common and give their coordinates.	3 marks
d)	Show that the graph of f_n for all n has two horizontal tangent lines and determine an equation of each of these tangent lines.	4 marks
e)	Determine the intervals where f_n is increasing or decreasing. Distinguish between even and odd values of n.	4 marks

c) Show that all the curves with equation $y = f_n(x)$ have two points in common and give their coordinates.	3 marks	

Solution 1: $\text{solve}(\mathbf{f2}(x)=\mathbf{f3}(x),x) \blacktriangleright x=0 \text{ or } x=1$

Solution 2: $f_m(x)=f_n(x) \Leftrightarrow x^m \cdot \mathbf{e}^{-x}=x^n \cdot \mathbf{e}^{-x} \Leftrightarrow x^m=x^n \Leftrightarrow x^n\left(x^{m-n}-1\right)=0$

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d) Show that the graph of f_n for all n has two horizontal tangent lines and determine an equation of each of these tangent lines.	4 marks

$$f_n'(x) = n \cdot x^{n-1} \cdot e^{-x} - x^n \cdot e^{-x} = x^{n-1} \cdot (n-x) \cdot e^{-x}$$

$$f_n'(x) = 0 \Leftrightarrow :$$

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e) Determine the intervals where f_n is increasing or decreasing. Distinguish between even and odd values of n.	4 marks	

Study the sign of $f_n'(x)$

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Part B - Calculator							
B1	a	Determine coord. Extrema pointsof interflection	1,0	4,0			5,0
Analysis	b	Detrmine volume solid totation x-axis	2,0	2,0			4,0
	c	Show points in common		2,0	1,0		3,0
Minimum 4 sub questions	d	Show graph has two horizontal tangent lines	2,0	2,0			4,0
	e	Determie intervals increasse/decrease- distingish odd even values of n	2,0	2,0			4,0
							0,0
							0,0
Maximum 8 sub questions							0,0
		S	7,0	12,0	1,0	0,0	20,0
		%	35,0	60,0	5,0	0,0	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

2. Let’s shift to a higher proportion of demanding tasks

Improving the exercise

What about investigating on n = 1?

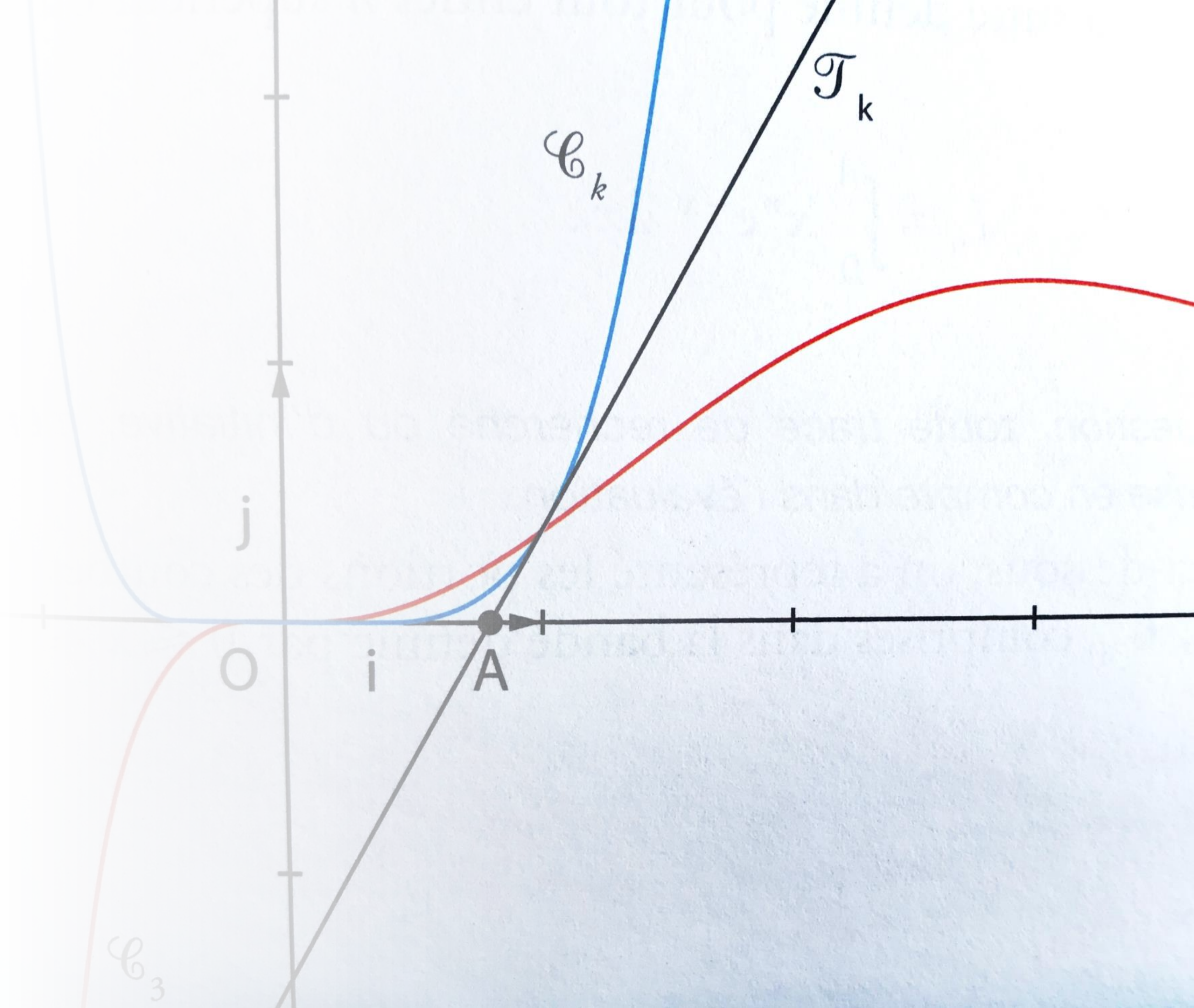
Consider the family of functions f_n defined by

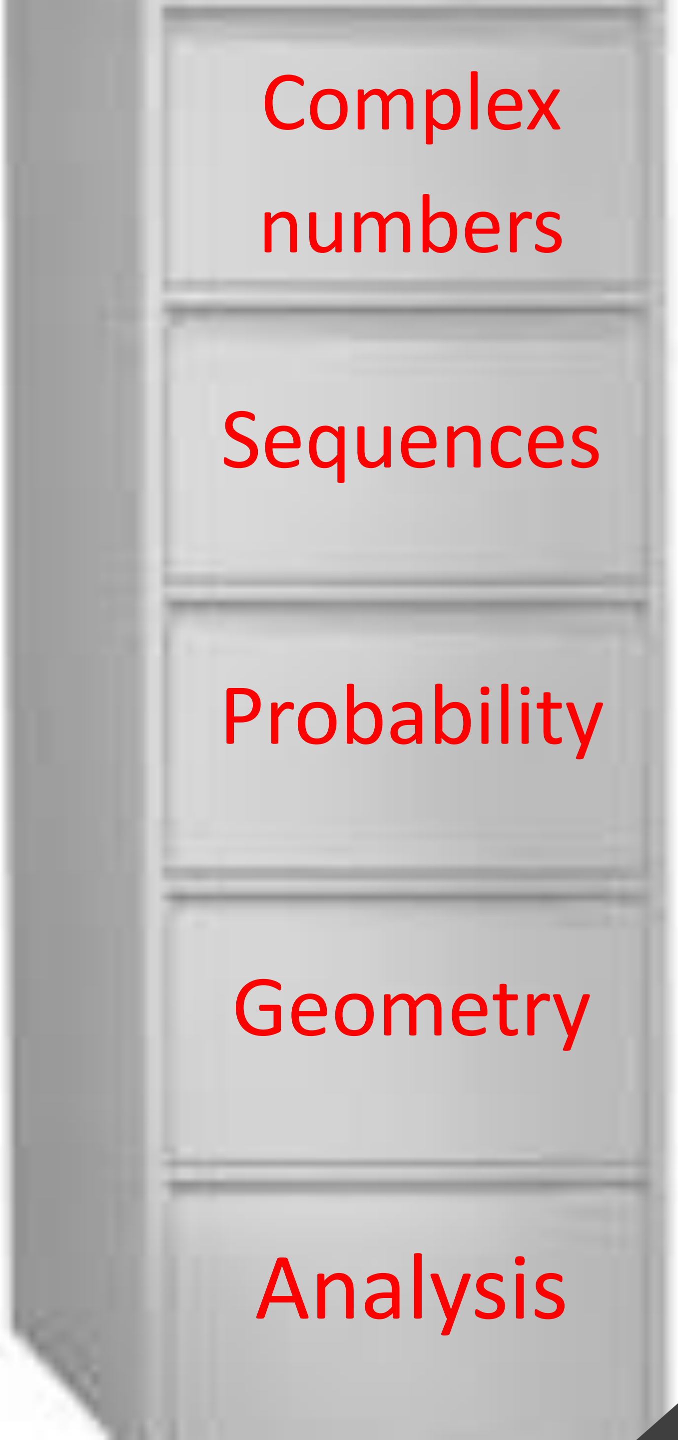
$$f_n(x) = x^n e^{-x}, \quad \text{where } n \in \{2, 3, 4, \ldots\}.$$

What type of question could we ask?

Higher level thinking

- On the graph, we have represented a curve C_k and the tangent T_k to the graph C_k in the point A where x=1.
- Knowing that the tangent intersects the x-axis in $A(\frac{4}{5};0)$, find k.

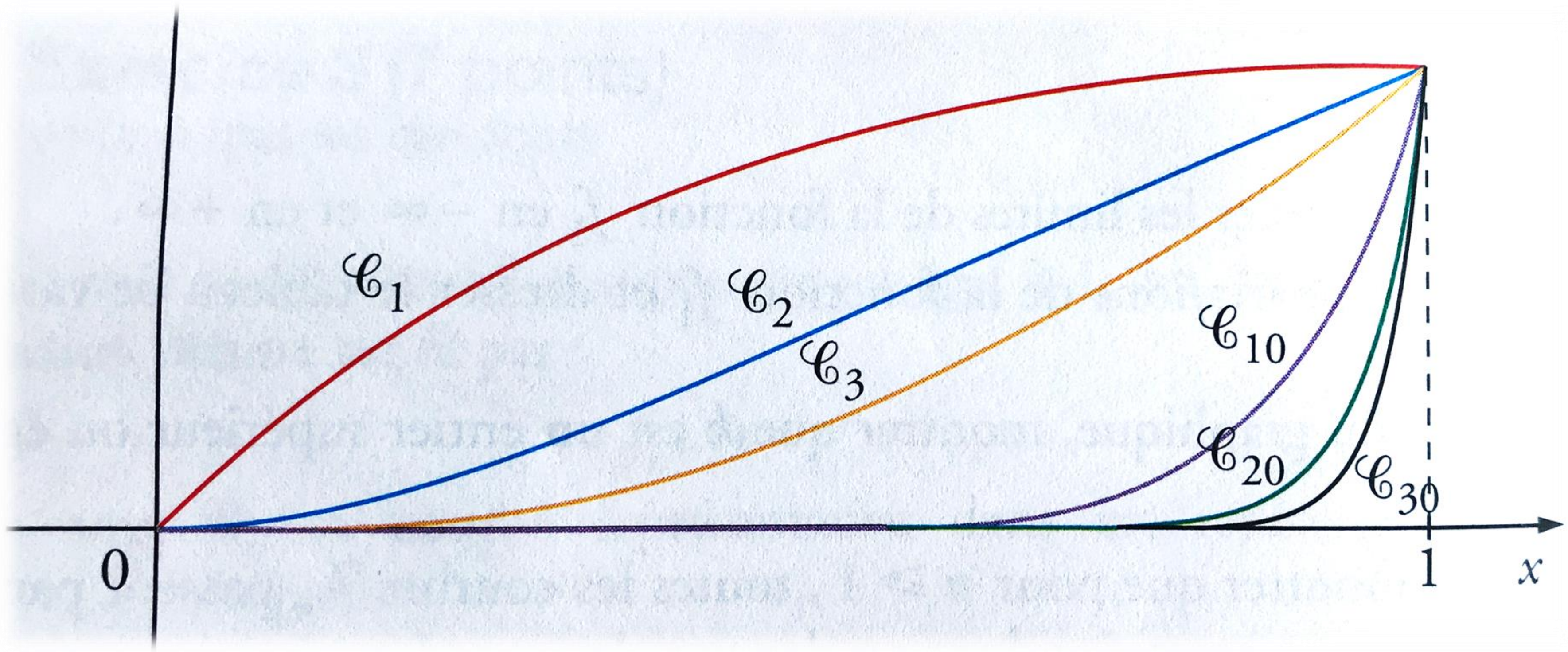




The 5P drawers!

A question that we could discuss now ...

Is it appropriate to lock the construction of knowledge in drawers?



Maybe in the future (near?, far?)

Consider the sequence given by: $I_n = \int_0^1 x^n e^{-x} dx \quad n \in \{1, 2, 3, ...\}$

The graph shows different curves restricted to $[0; 1]$

Formulate a hypothesis on the direction of variation of the sequence. Demonstrate this conjecture.

Deduce that the sequence is convergent.

Find: $\lim_{n \to +\infty} I_n$

My email address for further questions/suggestions/comments/...

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“The road to success is always under construction!”

Thanks again for listening for such a long time ...